About physical sense of Sachs form factors, amplitudes of a proton current with spin-flip and non spin-flip and violation of dipole dependence G_E and G_M from Q^2

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The Rosenbluth formula in the laboratory reference frame

Our notations for the process of ep elastic scattering in the Born approximation:

$$e(p_1) + p(q_1, s_1) \to e(p_2) + p(q_2, s_2)\,, \qquad (1)$$

$$M_{ep\to ep} = \overline{u}(p_2)\gamma^{\mu}u(p_1)\cdot\overline{u}(q_2)\Gamma_{\mu}(q^2)u(q_1)\;\frac{1}{q^2}\;, \qquad (2)$$

$$(J_p)_{\mu} = \overline{u}(q_2)\Gamma_{\mu}u(q_1)\,,\Gamma_{\mu}(q^2) = F_1\;\gamma_{\mu} + \frac{F_2}{4M}(\;\hat{q}\gamma_{\mu} - \gamma_{\mu}\hat{q}\;)\;, \qquad (3)$$

where $\overline{u}(p_i)u(p_i) = 2\,m_e,\; \overline{u}(q_i)u(q_i) = 2\,M\;(i = 1, 2)$, $p_i^2 = m_e^2,\; q_i^2 = M^2$, m_e and M – electron and proton mass, $q = q_2 - q_1$, s_1 and s_2 – spin 4-vectors for initial and final protons with the usual conditions of orthogonality and normalization: $s_1q_1 = s_2q_2 = 0$, $s_1^2 = s_2^2 = -1$. The Rosenbluth formula for $q_1 = (M, \vec{0})$ and $m_e = 0$ read as:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_2\cos^2(\theta_e/2)}{4E_1^3\sin^4(\theta_e/2)}\frac{1}{1+\tau}\left(G_E^2 + \frac{\tau}{\varepsilon}\,G_M^2\right)\;. \qquad (4)$$

$$G_E = F_1 + \frac{q^2}{4M^2}\,F_2\,,\; G_M = F_1 + F_2\,. \qquad (5)$$

Here $\tau = Q^2/4M^2$, $Q^2 = -q^2 = 4E_1E_2\sin^2(\theta_e/2)$, $\alpha = 1/137$ - fine structure constant, $\varepsilon^{-1} = 1 + 2(1+\tau)\tan^2(\theta_e/2)$.

The Rosenbluth formula in the arbitrary reference frame

[A.I. Akhiezer and V.B. Berestetsky, Quantum Electrodynamics, Nauka, Moscow, 1969, in Russian, eq.(34.3.3), page 475.]

The Rosenbluth formula in the arbitrary reference frame read as:

$$d\sigma = \frac{\alpha^2 do}{4w^2}\frac{1}{1+\tau}\,(\,G_E^2\,Y_I + \tau\,G_M^2\,Y_{II}\,)\,\frac{1}{q^4}\,, \tag{6}$$

$$Y_I = (p_+q_+)^2 + q_+^2 q^2, \quad Y_{II} = (p_+q_+)^2 - q_+^2(q^2+4m_e^2)\,,$$

$$p_+ = p_1 + p_2\,,\; q_+ = q_1 + q_2\,.$$

The Rosenbluth formulas in an arbitrary reference frame (6) as well as in the laboratory reference frame (4) are expressed only through the squares of the form factors (FFs) Sachs G_E^2 and G_M^2.

Physical meaning in decomposition of squares G_E^2 and G_M^2.

The cross section (4) summed over the polarizations of the initial and final protons can be represented as the sum of the cross sections without spin-flip ($\sigma^{\delta,\delta}$) and with spin-flip ($-\sigma^{\delta,\delta}$) of the initial proton:

$$\frac{d\sigma}{d\Omega} = \kappa \left(G_E^2 + \frac{\tau}{\varepsilon}\, G_M^2 \right) = \kappa\, (\sigma^{\delta,\delta} + \sigma^{-\delta,\delta}), \tag{7}$$

$$\sigma^{\delta,\delta} = G_E^2, \;\; \sigma^{-\delta,\delta} = \frac{\tau}{\varepsilon}\, G_M^2 \;. \tag{8}$$

where κ is the factor in front of the parentheses in Eq. (4).

The spin projections axis for initial and final proton are identical and coincide with the direction of the final proton momentum:

$$s_1 = (0, \vec{n}_2),\; s_2 = (|\vec{v}_2|, v_{20}\, \vec{n_2}), \vec{c}_1 = \vec{c_2} = \vec{n}_2 = \vec{q_2}/|\vec{q_2}|\,, \tag{9}$$

Spin 4-vectors s_1 and s_2 for protons with 4-momentum q_1, q_2 have the form:

$$s_i = (s_{i0}, \vec{s}_i), s_{0i} = \vec{v}_i\, \vec{c}_i, \vec{s}_i = \vec{c}_i + \frac{(\vec{c}_i \vec{v}_i)\, \vec{v}_i}{1 + v_{i0}}, v_i = (v_{i0}, \vec{v}_i) = q_i/M, i = 1, 2. \tag{10}$$

Physical meaning in decomposition of squares G_E^2 and G_M^2.

$$\sigma^{\delta,\delta} = G_E^2, \;\; \sigma^{-\delta,\delta} = \frac{\tau}{\varepsilon} G_M^2 \;. \tag{11}$$

The terms $\sigma^{\delta,\delta}$ and $\sigma^{-\delta,\delta}$ in Eq. (7), (11) are the cross sections without and with the spin-flip for the case where the initial and final protons are fully polarized in the direction of the motion of the final proton.

For the case when $\vec{c}_1 = \vec{n}_2$ and $\vec{c}_2 = \vec{n}_2$ we have $\sigma^{\delta,\delta}$.
For the case when $\vec{c}_1 = \vec{n}_2$ and $\vec{c}_2 = -\vec{n}_2$ we have $\sigma^{-\delta,\delta}$.

To prove the relation (11) there are three ways:

- Using the standard method calculation for QED processes cross sections.
- With help of F. Halzen and A. Martin book "Quarks and leptons. An Introductory Course in Modern Particle Physics", Page 178, 1984 (in English), Page 214, 1987 (in Russian).
- Using the method for calculating QED processes matrix elements in the so-called “diagonal spin basis” (DSB).

Standard method for calculation QED cross sections

[M.Galynskii, E.Kuraev, Yu.Bystritskiy, JETP Lett. 88 (2008) 481-486, Pisma Zh.Eksp.Teor.Fiz. 88 (2008) 555-560, arXiv: 0805.0233 [hep-ph]].

$$\sigma \sim |M_{ep\to ep}|^2 = |\overline{u}(p_2)\gamma^\mu u(p_1)\cdot\overline{u}(q_2)\Gamma_\mu(q^2)u(q_1)|^2\,,$$
$$\sigma(s_1,s_2) \sim \mathrm{Tr}(\tau_2^e\gamma^\mu\tau_1^e\gamma^\nu)\cdot\mathrm{Tr}(\tau_2^p\,\Gamma_\mu\tau_1^p\,\overline{\Gamma}_\nu)\,, \quad (12)$$

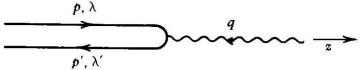
$$\tau_1^e = \frac{1}{2}(\hat{p_1}+m_e),\ \tau_2^e = \frac{1}{2}(\hat{p_2}+m_e),$$
$$\tau_1^p = \frac{1}{2}(\hat{q_1}+M)(1-\delta_1\gamma_5\hat{s}_1),\ \tau_2^p = \frac{1}{2}(\hat{q_2}+M)(1-\delta_2\gamma_5\hat{s}_2),$$
$$s_1 = (0,\vec{n_2}),\ s_2 = (|\vec{v_2}|, v_{20}\,\vec{n_2}), \vec{c_1} = \vec{c_2} = \vec{n}_2 = \vec{q_2}/|\vec{q_2}|\,, \quad (13)$$

$$\sigma^{\delta,\delta} = \sigma(s_1,s_2) = \sigma(\vec{c}_1 = \vec{c}_2 = \vec{n}_2) \sim (1+\delta_1\delta_2)G_E^2\,, \quad (14)$$
$$\sigma^{-\delta,\delta} = \sigma(s_1,-s_2) = \sigma(\vec{c}_1 = -\vec{c}_2 = \vec{n}_2) \sim (1-\delta_1\delta_2)G_M^2 \quad (15)$$
$$\sigma = \sigma^{\delta,\delta} + \sigma^{-\delta,\delta}\,. \quad (16)$$

Here we proposed a new method for measuring the Sachs FFs. They can be determined separately and independently by direct measurements of the cross sections without and with spin-flip of the initial proton, which should be at rest and fully polarized in the direction of the motion of the scattered proton.

Exercise 8.7 at page 178 from F. Halzen and A. Martin book "Quarks and leptons", 1984 (page 214, 1987, in Russian)

Evaluate $J^{\mu}(0) \equiv (\rho, \mathbf{J})$ in the Breit frame ($\mathbf{p}' = -\mathbf{p}$). There is no energy



transferred to the proton in this frame, and it behaves as if it had bounced off a brick wall, see Fig. 8.3. If the z axis is chosen along $\mathbf{p}$ and helicity spinors are used, show that

$$\rho = 2Me\,G_E(q^2) \qquad \text{for } \lambda = -\lambda',$$

$$J_1 \pm iJ_2 = \mp 2|\mathbf{q}|e\,G_M(q^2) \qquad \text{for } \lambda = \lambda' = \mp\tfrac{1}{2},$$

and that all other matrix elements are zero; λ and λ' denote the initial and final proton helicities, respectively.

From this picture, we see that in the Breit-system a transition with (without) a change in the sign of helicity is the transition without (with) spin-flip of the proton:

$$J_{\mu}^{-\lambda,\lambda} = J_{\mu}^{\delta,\delta} = 2\,e\,M\,G_E\,(b_0)_{\mu}\,, b_0 = (1,0,0,0),\; b_0^2 = 1, \qquad (17)$$

$$J_{\mu}^{\lambda,\lambda} = J_{\mu}^{-\delta,\delta} = -2e\,\delta|\vec{q}|\,G_M\,(b_{\delta})_{\mu}\,, |\vec{q}| = \sqrt{Q^2}, b_{\delta} = b_1 + i\delta b_2, \qquad (18)$$

$$b_1 = (0,1,0,0), b_2 = (0,0,1,0), \delta = \pm 1.$$

Exercise 8.7 at page 178 from F. Halzen and A. Martin book "Quarks and leptons", 1984 (page 214, 1987, in Russian)

Below we have dropped the factor e:

$$J_\mu^{\delta,\delta} = 2\,M\,G_E\,(b_0)_\mu\,, \tag{19}$$

$$J_\mu^{-\delta,\delta} = -2\,M\delta\sqrt{\tau}\,G_M\,(b_\delta)_\mu\,, b_\delta = b_1 + i\delta b_2,\; \delta = \pm 1, \tag{20}$$

$$b_0 = (1,0,0,0)\,, b_1 = (0,1,0,0), b_2 = (0,0,1,0), b_3 = (0,0,0,1)\,.$$

In the Breit system where $q_1 = (q_0, -\vec{q}), q_2 = (q_0, \vec{q})$, the spin states of the initial and final protons are helical, so they spin four-vectors s_1 и s_2 have the form:

$$s_1 = (-|\vec{v}|, v_0\vec{n}_2),\; s_2 = (|\vec{v}|, v_0\vec{n}_2)\,, \vec{n}_2 = \vec{q_2}/|\vec{q_2}|\,. \tag{21}$$

Let us make transition from Breit system to an arbitrary reference frame or for example to the rest frame of the initial proton. For this purpose we need to write the four basic vectors b_A in the covariant form. We will construct 4-vectors b_A through the 4-momenta of participating in the reaction particles.

Exercise 8.7 at page 178 from F. Halzen and A. Martin book "Quarks and leptons", 1984 (page 214, 1987, in Russian)

4-vectors b_0 and b_3 can be written as the normalized per unit the sum and difference between the momenta of final and initial protons:

$$b_0 = \frac{q_+}{\sqrt{q_+^2}}, \;\; q_+ = q_1 + q_2 = (2q_0, \vec{0}), \Rightarrow b_0 = (1, \vec{0}), \quad (22)$$

$$b_3 = \frac{q_-}{\sqrt{-q_-^2}}, \;\; q_- = q_2 - q_1 = (0,0,0,2q), \Rightarrow b_3 = (0,0,0,1), \quad (23)$$

$$(b_1)_\mu = \varepsilon_{\mu\nu\kappa\sigma} b_0^\nu b_3^\kappa b_2^\sigma, \; (b_2)_\mu = \varepsilon_{\mu\nu\kappa\sigma} b_0^\nu b_3^\kappa p_1^\sigma / \rho \, , b_\delta = b_1 + i\delta b_2. \quad (24)$$

Here, $\varepsilon_{\mu\nu\kappa\sigma}$ is the Levi-Civita tensor ($\varepsilon_{0123} = -1$), ρ is determined from the normalization conditions $b_2^2 = -1$, p_1 is 4-momentum of initial electron.
The proton current matrix elements (19), (20) by using (22)-(24) can be write in the form

$$J_\mu^{\delta,\delta} = 2\, M\, G_E\, (b_0)_\mu \, , \quad (25)$$

$$J_\mu^{-\delta,\delta} = -2\, M \delta \sqrt{\tau}\, G_M\, (b_\delta)_\mu \, . \quad (26)$$

These expressions are valid in arbitrary reference frame.

Exercise 8.7 at page 178 from F. Halzen and A. Martin book "Quarks and leptons", 1984 (page 214, 1987, in Russian)

With the help of the matrix elements of the proton current (25), (26) calculation probability of the process $ep \to ep$ is reduced to calculation of the trivial trace:

$$|\,T\,|^2 = \frac{4M^2}{q^4}\,\frac{1}{8}\sum_{\delta} Tr(G_E^2(\hat{p}_2+m)\hat{b}_0(\hat{p}_1+m)\hat{b}_0 + \qquad (27)$$

$$+\tau\, G_M^2(\hat{p}_2+m)\hat{b}_\delta(\hat{p}_1+m)\hat{b}_\delta^*)\,,$$

where $b_\delta^* = b_{-\delta} = b_1 - i\delta b_2, \delta = \pm 1$. Calculation of $|\,T\,|^2$ leads to an expression for the cross section, which coincides with (6):

$$d\sigma = \frac{\alpha^2 do}{4w^2}\frac{1}{1+\tau}\,(\,G_E^2\,Y_I + \tau\,G_M^2\,Y_{II}\,)\,\frac{1}{q^4}\,, \qquad (28)$$

$$Y_I = (p_+q_+)^2 + q_+^2 q^2, \qquad Y_{II} = (p_+q_+)^2 - q_+^2(q^2+4m_e^2)\,,$$

$$p_+ = p_1 + p_2\,,\; q_+ = q_1 + q_2\,.$$

Thus, the differential cross section for the $ep \to ep$ process naturally splits into the sum of two terms containing only the squares of the Sachs FFs and corresponding to the contribution of transition without ($\sim G_E^2$) and with ($\sim G_M^2$) proton spin-flip.

Diagonal spin basis (DSB)

[S. Sikach, Izvestia AN BSSR, s.f.-m.n, № 2, 84 (1984)]
In the diagonal spin basis (DSB) spin 4-vectors s_1 and s_2 of protons with 4-momenta q_1 and q_2 ($s_1 q_1 = s_2 q_2 = 0, s_1^2 = s_2^2 = -1$) have the form:

$$s_1 = -\frac{(v_1 v_2) v_1 - v_2}{\sqrt{(v_1 v_2)^2 - 1}}\ , \ \ s_2 = \frac{(v_1 v_2) v_2 - v_1}{\sqrt{(v_1 v_2)^2 - 1}}\ , v_1 = \frac{q_1}{M},\ v_2 = \frac{q_2}{M}, \tag{29}$$

The spin vectors (29) obviously do not change under transformations of the Lorentz little group common to particles with 4-momenta q_1 and q_2: $L_{q_1,q_2} q_1 = q_1$, $L_{q_1,q_2} q_2 = q_2$. It is a one-parameter subgroup of the rotation group SO_3 with axis whose direction is determined by the 3-vector [F.I. Fedorov, Theor.Math.Fiz, **2**, № 3, 343 (1970)]:

$$\vec{a} = \vec{q}_1 / q_{10} - \vec{q}_2 / q_{20}\ . \tag{30}$$

The direction of $\vec{a}$ (30) have property that the projections of the spins of both particles on it simultaneously have definite values. Therefore, the DSB naturally makes it possible to describe the spin states of systems of any two particles by means of the spin projections on the common direction given by the 3-vector (30).

Diagonal spin basis (DSB)

Since vector $\vec{a}$ (30) is the difference of two vectors and the geometrical image of the difference of two vectors is the diagonal of a parallelogram, hence the name "diagonal spin basis"given by academician F.I.Fedorov.
Consider the realization DSB in the rest frame of the initial proton, where $q_1 = (M, \vec{0})$. Here $\vec{a}$ (30) equal $\vec{a} = \vec{n}_2 = \vec{q_2}/|\vec{q_2}|$, i.e. common direction for spin projection is the direction of the motion of the final proton, thus this final proton polarization state is a helicity and spin 4-vectors s_1 и s_2 (29) have the form:

$$s_1 = (0, \vec{n}_2),\; s_2 = (|\vec{v}_2|, v_{20}\, \vec{n_2}), \vec{c}_1 = \vec{c_2} = \vec{n}_2 = \vec{q_2}/|\vec{q_2}|\,, \tag{31}$$

axis of spin projections $\vec{c}_1$ and $\vec{c_2}$ is coincide with the direction of the final proton. Breit system, where $\vec{q_2} = -\vec{q_1}$, is a special case of DSB. In the Breit system where $q_1 = (q_0, -\vec{q}), q_2 = (q_0, \vec{q})$, the spin states of the initial and final protons are helicity, so they spin 4-vectors s_1 и s_2 in DSB have the form:

$$s_1 = (-|\vec{v}|, v_0\vec{n}_2),\; s_2 = (|\vec{v}|, v_0\vec{n}_2)\,, \vec{n}_2 = \vec{q_2}/|\vec{q_2}|\,. \tag{32}$$

Note that the transition from Breit system to rest frame of the initial proton 4-spin vectors (32) go into expressions (31).

Spin operators in the DSB

[M. Galynskii and S. Sikach, Fiz. Elem. Chast. Atom. Yadra, **29**, 1133-1193, (1998), e-print: hep-ph/9910284]

In the DSB spin operators for initial and final proton coincide and have the same form:

$$\sigma = \sigma_1 = \sigma_2 = \gamma^5 \hat{s_1}\hat{v_1} = \gamma^5 \hat{s_2}\hat{v_2} = \gamma^5 \hat{b}_0 \hat{b}_3 = i\hat{b}_1\hat{b}_2 \;, \tag{33a}$$

$$\sigma^{\pm\delta} = \sigma_1^{\pm\delta} = \sigma_2^{\pm\delta} = -i/2\gamma^5 \hat{b}_{\pm\delta},\; b_{\pm\delta} = b_1 \pm i\delta b_2 \;,\; \delta = \pm 1 \;, \tag{33b}$$

$$\sigma u^\delta(q_i) = \delta u^\delta(q_i) \;,\; \sigma^{\pm\delta} u^{\mp\delta}(q_i) = u^{\pm\delta}(q_i). \tag{33c}$$

The set of unit 4-vectors b_0, b_1, b_2, b_3 is an orthonormal basis of 4-vectors b_A, $b_A b_B = g_{AB}$ $(A, B = 0, 1, 2, 3)$:

$$(b_1)_\mu = \varepsilon_{\mu\nu\kappa\sigma} b_0^\nu b_3^\kappa b_2^\sigma,\; (b_2)_\mu = \varepsilon_{\mu\nu\kappa\sigma} b_0^\nu b_3^\kappa p_1^\sigma / \rho \,, b_3 = \frac{q_-}{\sqrt{-q_-^2}},\; b_0 = \frac{q_+}{\sqrt{q_+^2}} \;, \tag{34}$$

where $q_- = q_2 - q_1$, $q_+ = q_2 + q_1$, $\varepsilon_{\mu\nu\kappa\sigma}$ is the Levi-Civita tensor ($\varepsilon_{0123} = -1$), ρ is determined from the normalization conditions $b_1^2 = b_2^2 = b_3^2 = -b_0^2 = -1$.

Calculation of QED matrix elements in the DSB

[M. Galynskii and S. Sikach, Fiz. Elem. Chast. Atom. Yadra, **29**, 1133-1193, (1998), e-print: hep-ph/9910284]:
A matrix elements of QED processes have a standard form

$$M^{\pm\delta,\delta} = \overline{u}^{\pm\delta}(q_2) Q u^{\delta}(q_1) \,, \tag{35}$$

where Q is the interaction operator, and $u^{\delta}(q_1)$ and $u^{\pm\delta}(q_2)$ are the bispinors of the initial and final states, with $\overline{u}^{\delta}(q_i)\, u^{\delta}(p_i) = 2M$, $q_i^2 = M^2$, $(i = 1, 2)$.
In our covariant approach the calculation of matrix elements of the form (35) reduces to computation the trace from the product of Dirac operators:

$$M^{\pm\delta,\delta} = Tr(P_{21}^{\pm\delta,\delta} Q) \,,\; P_{21}^{\pm\delta,\delta} = u^{\delta}(q_1)\, \overline{u}^{\;\pm\delta}(q_2) \,. \tag{36}$$

The operators $P_{21}^{\pm\delta,\delta}$ determine the structure of the spin dependence of the matrix elements (35) in the case of transitions without spin-flip ($P_{21}^{\delta,\delta}$) and with spin-flip ($P_{21}^{-\delta,\delta}$). They have the form:

$$P_{21}^{\delta,\delta} = (\hat{q}_1 + M)\, \hat{b}_\delta\, \hat{b}_0\, \hat{b}_\delta^*/4 \,, \tag{37}$$

$$P_{21}^{-\delta,\delta} = \delta(\hat{q}_1 + M)\, \hat{b}_\delta\, \hat{b}_3/2 \,, \tag{38}$$

where $b_\delta^* = b_1 - i\delta b_2$.

The matrix elements of the proton current in the DSB

[S. Sikach, Izvestia AN BSSR, s.f.-m.n, № 2, 84 (1984)]
The process of elastic ep scattering

$$e(p_1) + p(q_1, s_1) \to e(p_2) + p(q_2, s_2)\,, \tag{39}$$

where s_1 and s_2 are spin 4-vectors for initial and final protons in the DSB (29). Matrix elements (amplitudes) for proton current defined as:

$$(J_p^{\pm\delta,\delta})_\mu = \overline{u}^{\pm\delta}(q_2)\Gamma_\mu(q^2)u^\delta(q_1)\,,\;\; \Gamma_\mu(q^2) = F_1\,\gamma_\mu + \frac{F_2}{4M}(\,\hat{q}\gamma_\mu - \gamma_\mu\hat{q}\,)\,, \tag{40}$$

they were calculated by S.Sikach (1984):

$$(J_p^{\delta,\delta})_\mu = 2G_E M(b_0)_\mu\,, (J_p^{-\delta,\delta})_\mu = -2\delta M\sqrt{\tau}G_M(b_\delta)_\mu\;, \tag{41}$$

$$G_E = F_1 + \frac{q^2}{4M^2}\,F_2\,,\; G_M = F_1 + F_2\;. \tag{42}$$

For the point particles with mass m_q the amplitude of the currents have the form

$$(J_q^{\delta,\delta})_\mu = 2\,m_q\,(b_0)_\mu\,, (J_q^{-\delta,\delta})_\mu = -2\,m_q\,\delta\sqrt{\tau_q}\,(b_\delta)_\mu\;\;. \tag{43}$$

where $\tau_q = Q_q^2/4m_q^2$.

The matrix elements of the proton current in the DSB

Since $|b_0| = 1$ and $|b_\delta b_\delta^*| = 2$ and they are does not depend on Q^2, then from the (41), (43), we can easily obtain the dependence on Q^2 for (absolute) values of the matrix elements of proton currents $J_p^{\pm\delta,\delta}$ and point particles $J_q^{\pm\delta,\delta}$:

$$J_p^{\delta,\delta} \sim 2\,M\,G_E,\ J_p^{-\delta,\delta} \sim 2\,M\,\sqrt{\tau}\,G_M\,, \tag{44}$$

$$J_q^{\delta,\delta} \sim 2\,m_q\,,\ J_q^{-\delta,\delta} \sim 2\,m_q\,\sqrt{\tau_q}\,. \tag{45}$$

Note that the factorization of $2M$ and $2m_q$ in the expressions (44), (45) is caused by the normalization bispinors $\bar{u}_i u_i = 2m_i$. Below during the computation is more convenient to use the normalization of $\bar{u}_i u_i = 1$, and instead of (44), (45) we will use the expressions:

$$J_p^{\delta,\delta} \sim\ G_E, J_p^{-\delta,\delta} \sim\ \sqrt{\tau}\,G_M\,, \tag{46}$$

$$J_q^{\delta,\delta} \sim 1\,,\ \ J_q^{-\delta,\delta} \sim \sqrt{\tau_q}\,. \tag{47}$$

Expressions (46), (47) we will be used below to explain the violation of the dipole dependence FFs G_E and G_M from Q^2. Here we will use the model in which the proton consists from three point quarks with equal masses m_q, and the matrix element of the proton current is the product of three quark current amplitudes of the form: $J_q^{\delta,\delta} \sim 1$, $J_q^{-\delta,\delta} \sim \sqrt{\tau_q}$. It will be show that the dipole dependence arises when the amplitude of quark currents without spin-flip are dominated.

About the violation of dipole dependence G_E and G_M

[M.Galynskii, E.Kuraev, Pisma Zh. Eksp. Teor. Fiz., **96**, № 1, 8-13 (2012).]

Proton transition without spin-flip can be realized in two ways: 1) all three quark are not spin flip, 2) two quark spins flips, and the third does not. We denote the number such means as $n_q^{\delta,\delta} = [0,2]$ according to the number of quarks flipping the spin (either single or two).

Proton spin-flip can also be realized in two ways: 1) one quark spin flips, and two others are not, and 2) all three quark spin flips. We denote the number such means as $n_q^{-\delta,\delta} = [1,3]$ according to the number of quarks flipping the spin (or one overturned, or all three). Thus, there are only four combinations to be considered:

$$n_q^{\delta,\delta} \times n_q^{-\delta,\delta} = [0,2] \times [1,3] = (0,1) + (0,3) + (2,1) + (2,3)\,. \tag{48}$$

Of these the first, (0.1) corresponds to the dipole dependence of FFs G_E and G_M from Q^2, when in the proton transition without spin-flip none of the quark not spin flips (the first number in brackets is zero), and proton spin-flip due to spin-flip of one quark (the second number in brackets is equal to 1).

For the sets (0.1) and (2.3) in (48) we get $G_E/G_M \sim 1$, for a set of (0.3): $Q^2 G_E/G_M \sim 4M^2$, and for a set of (2,1): $Q^2 G_M/G_E \sim 4M^2$.

About the violation of dipole dependence G_E and G_M

Set (0,1). The dipole dependence G_E and G_M from Q^2, $G_e/G_M \sim 1$.

We use for the amplitudes of protons and point-like particles currents next expressions:

$$J_p^{\delta,\delta} \sim G_E, J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M \,, \tag{49}$$

$$J_q^{\delta,\delta} \sim 1\,, \quad J_q^{-\delta,\delta} \sim \sqrt{\tau_q}\,. \tag{50}$$

We will use the model in which the proton consists of three point quarks with the same mass and the proton current amplitude is the product of the three quark currents amplitudes, which is conveniently represented by the following diagram:

$$J_p^{\delta,\delta} \sim G_E \quad \sim \quad \begin{matrix} + & \to\to * \to\to\to\to & + \\ - & \to\to\to * \to\to\to & - \\ + & \to\to\to\to * \to\to & + \end{matrix} \quad \text{non spin-flip}, \tag{51}$$

$$J_p^{\delta,\delta} \sim G_E \quad \sim \quad 1 \times 1 \times 1 \times \frac{1}{Q^4}\,, \quad \Rightarrow \quad G_E \sim \frac{1}{Q^4}\,, \tag{52}$$

where the factors "$1''$ correspond to transitions without spin-flip (see 50) of three point quarks with a mass of m_q, and Q^4 in the denominator arises because of two gluon propagators.

About the violation of dipole dependence G_E and G_M

Set (0,1). The dipole dependence G_E and G_M on Q^2, $G_e/G_M \sim 1$.

$$J_p^{\delta,\delta} \sim G_E, \quad J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M\,, \tag{53}$$

$$J_q^{\delta,\delta} \sim 1\,, \quad J_q^{-\delta,\delta} \sim \sqrt{\tau_q}\,. \tag{54}$$

$$J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M \quad \sim \quad \begin{array}{ccc} + & \to\to * \to\to\to\to & - \\ - & \to\to\to * \to\to\to & - \\ + & \to\to\to\to * \to\to & + \end{array} \quad \text{spin-flip}\,. \tag{55}$$

The diagram (55) corresponds to a transition in which the spin of the top quark is flipped, while the bottom two is not, that generally corresponds to the transition of a proton with spin flip. As a result, we have

$$J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M \sim \sqrt{\tau_q} \times 1 \times 1 \times \frac{1}{Q^4} \Rightarrow G_M \sim \frac{\sqrt{\tau_q}}{\sqrt{\tau}} \frac{1}{Q^4}\,. \tag{56}$$

We assume that $m_q = M/3$ and $Q_q = Q/3$. This leads to the equality $\sqrt{\tau_q}/\sqrt{\tau} = 1$. (Below, we will always assume $\sqrt{\tau_q} = \sqrt{\tau}$). As a result, we have:

$$G_E \sim \frac{1}{Q^4},\, G_M \sim \frac{1}{Q^4}\,,\, \frac{G_E}{G_M} \sim 1\,. \tag{57}$$

About the violation of dipole dependence G_E and G_M

Set (0,3). The non-dipole dependence, $G_E/G_M \sim 4M^2/Q^2$.

$$J_p^{\delta,\delta} \sim G_E, \quad J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M\,, \tag{58}$$

$$J_q^{\delta,\delta} \sim 1\,, \qquad J_q^{-\delta,\delta} \sim \sqrt{\tau_q}\,. \tag{59}$$

For violations of the dipole dependence it is necessary that the number of spin flipped quarks was not minimal as in the case of set (0.1). Here we consider a set of (0.3), when the contribution to the $J_p^{-\delta,\delta}$ give the spin-flip transitions of the three quarks. This can only happen in the case when transferred to the proton momenta Q^2 high. Let as write the equality similar to (52) and (56):

$$J_p^{\delta,\delta} \sim G_E \sim 1 \times 1 \times 1 \times \frac{1}{Q^4}\,, \tag{60}$$

$$J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M \sim \sqrt{\tau} \times \sqrt{\tau} \times \sqrt{\tau} \times \frac{1}{Q^4}\,, \tag{61}$$

$$G_E \sim \frac{1}{Q^4}\,,\; G_M \sim \frac{\tau}{Q^4}\,, \frac{G_E}{G_M} \sim \frac{1}{\tau} \sim \frac{4M^2}{Q^2}\,, \tag{62}$$

$$Q^2\,\frac{G_E}{G_M} \sim 4M^2 = const\,. \tag{63}$$

Consequently, at $Q^2 > 4M^2$ the ratio $G_E/G_M < 1$.

About the violation of dipole dependence G_E and G_M

Set (2,3). The ratio $G_E/G_M \sim 1$, G_E and G_M dependence on Q^2 is not dipole.

$$J_p^{\delta,\delta} \sim G_E, \quad J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M\,,$$
$$J_q^{\delta,\delta} \sim 1\,, \quad J_q^{-\delta,\delta} \sim \sqrt{\tau_q}\,.$$

Consider a spin combination (2.3) in the set (48), which is realized for contributions to $J_p^{\delta,\delta}$ transitions of two quarks with spin flip, and for contributions to $J_p^{-\delta,\delta}$ transitions of all three quarks with spin flip. For this case, we have

$$J_p^{\delta,\delta} \sim G_E \sim \sqrt{\tau} \times \sqrt{\tau} \times 1 \times \frac{1}{Q^4}\,, \tag{64}$$

$$J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M \sim \sqrt{\tau} \times \sqrt{\tau} \times \sqrt{\tau} \times \frac{1}{Q^4}\,. \tag{65}$$

$$G_E \sim \frac{\tau}{Q^4}\,, \quad G_M \sim \frac{\tau}{Q^4}\,, \quad \frac{G_E}{G_M} \sim 1\,. \tag{66}$$

Consequently, the ratio of form factors of $G_E/G_M \sim 1$ behaves like a dipole model. However, the dependence of $G_E \sim 1/(4M^2Q^2)$ and $G_M \sim 1/(4M^2Q^2)$ is not a dipole ($G_E \sim 1/Q^4$, $G_M \sim 1/Q^4$) and was not observed in an experiments.

Set (2,1). The dependence $G_E/G_M \sim Q^2/4M^2$.

$$J_p^{\delta,\delta} \sim G_E, \quad J_p^{-\delta,\delta} \sim \sqrt{\tau}\, G_M\,,$$
$$J_q^{\delta,\delta} \sim 1\,, \quad J_q^{-\delta,\delta} \sim \sqrt{\tau_q}\,.$$

Consider a spin combination (2,1) in the set (48), which realized at the contribution to $J_p^{\delta,\delta}$ by transitions of two quarks with spin-flip, and for contributions to $J_p^{-\delta,\delta}$ by the transition with spin-flip only of one quark. Acting is completely analogous to the above, it is easy to see that for a set of (2,1) G_E and G_M have the form

$$G_E \sim \frac{\tau}{Q^4}\,,\; G_M \sim \frac{1}{Q^4}\,, \tag{67}$$

i.e. ratio of G_E/G_M behaves as

$$\frac{G_E}{G_M} \sim \tau \sim \frac{Q^2}{4M^2}\,,\; Q^2\,\frac{G_M}{G_E} \sim 4M^2 = const\,. \tag{68}$$

The spin parameterization for the ratio G_E/G_M.

The amplitudes of the proton current without ($J_p^{\delta,\delta}$) and with spin-flip ($J_p^{-\delta,\delta}$) of the proton can be expressed as linear combinations:

$$J_p^{\delta,\delta} = \alpha_0\, J_q^{\delta,\delta} J_q^{-\delta,-\delta} J_q^{\delta,\delta} + \alpha_2\, J_q^{-\delta,\delta} J_q^{\delta,-\delta} J_q^{\delta,\delta}, \tag{69}$$

$$J_p^{-\delta,\delta} = \beta_1\, J_q^{-\delta,\delta} J_q^{\delta,\delta} J_q^{-\delta,-\delta} + \beta_3\, J_q^{-\delta,\delta} J_q^{\delta,-\delta} J_q^{-\delta,\delta}, \tag{70}$$

where the physical meaning of the coefficients α_0, α_2 и β_1, β_3 are defined by their indices, which determine the number of quarks with spin-flip, contributing to the proton transitions without and with spin-flip. With help of (69), (70) we can easily obtain a general expression for the ratio G_E/G_M:

$$\frac{G_E}{G_M} = \frac{\alpha_0\ + \alpha_2\,\tau}{\beta_1\ + \beta_3\,\tau}\ . \tag{71}$$

This expression can be the basis for the spin parameterization and fitting of experimental data on measurement of the ratio G_E/G_M. Due to the requirement of the dipole depending realization at $\tau \ll 1$ α_0 and β_1 in (71) must be close to unity: $\alpha_0 \sim 1$ and $\beta_1 \sim 1$. Take account this remark and expand (71) in the series for $\tau \ll 1$. As result we get the linear decreasing law for ratio G_E/G_M:

$$\frac{G_E}{G_M} \sim 1 - \frac{(\beta_3 - \alpha_2)}{4M^2}\ Q^2\ . \tag{72}$$

Notes: what we have in the literature about $J_p^{\delta,\delta}$ and $J_p^{-\delta,\delta}$

For $\Gamma_\mu(q^2)$ of the proton there are two equivalent expressions:

$$\Gamma_\mu(q^2) = F_1\ \gamma_\mu + \frac{F_2}{4M}(\ \hat{q}\gamma_\mu - \gamma_\mu\hat{q}\ )\ , \tag{73}$$

$$\Gamma_\mu(q^2) = G_M\gamma_\mu - \frac{(q_1+q_2)_\mu}{2m}\,F_2\,, \tag{74}$$

they are linked by Gordon transformation and give the same result for proton current matrix elements in the DSB:

$$(J_p^{\delta,\delta})_\mu = 2G_E M(b_0)_\mu\,,(J_p^{-\delta,\delta})_\mu = -2\delta M\sqrt{\tau}G_M(b_\delta)_\mu\ . \tag{75}$$

In the literature, however, based only on the explicit form for $\Gamma_\mu(q^2)$ (73), (74) without calculating incorrectly states that at large q_1 and q_2 the Dirac FF F_1 is responsible to the proton transition without helicity-flip, and Pauli FF F_2 is responsible for transition with helicity-flip. Most likely that this statements was started from work [G. Lepage, S. Brodsky. Phys.Rev. **D22**, 2157 (1980)].
In fact, for transitions with a helicity-flip (without spin-flip) is responsible G_E, but not F_2. And for the transition without helicity-flip (with spin-flip) is responsible G_M, but not F_1.

Conclusion

The fundamental physical meaning of electric and magnetic FFs of G_E and G_M consists in that they define the matrix elements of the proton current in the case of transitions without ($J_p^{\delta,\delta}$) and with spin-flip ($J_p^{-\delta,\delta}$) of the proton, respectively. They have already factored in the amplitudes $J_p^{\delta,\delta}$, $J_p^{-\delta,\delta}$ of the proton current.

On the quark level we used a model in which the proton is composed from three point quarks with equal masses, and the matrix element of the proton current is the product of three quark currents amplitude of the form $J_q^{\delta,\delta} \sim 1$, $J_q^{-\delta,\delta} \sim \sqrt{\tau}$.

It is shown that the dipole dependence is originated when the proton spin-flip is realized through spin-flip only of one from three quarks. Violations of the same is due to its contributions to $J_p^{\delta,\delta}$ transitions with spin-flip of two quarks, or contribution to $J_p^{-\delta,\delta}$ spin-flip transitions all three quarks in the proton.

Therefore, the measurement of the ratio G_E/G_M can "look inside the proton" and to determine the number of flipping and non flipping spin of the quarks.

Thanks

THANK FOR YOUR ATTENTION